

So what is simple harmonic motion and what is the defining equation?

Consider the force acting on a body executing SHM (a bob oscillating on a horizontal spring, for example):

Bob is to the right of equilibrium  
 spring is stretched  
 displacement is to the right (+)  
 force is to the left (-)

Bob is at equilibrium  
 no displacement, force is zero

Bob is left of equilibrium  
 spring is compressed  
 displacement is to the left (-)  
 force is to the right (+)

Recall Hooke's Law  $\rightarrow$  force is proportional to the displacement from equilibrium.  
 force is in the opposite direction of displacement.

$F_{net} = -kx$  (negative)  
 Acceleration of a body executing SHM  
 where  $k$  is the spring or force constant.

Recall Newton's 2nd Law:  $F_{net} = ma$  where  $a$  is the acceleration.

$ma = -kx$   
 $a = -\frac{k}{m}x$   
 acceleration is directly proportional to the displacement but is in the opposite direction to the displacement (i.e. towards the equilibrium)

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(a) Stretched compared with (b)  
 $x$  is positive (to right)  
 $F$  is negative (to left)  
 $a$  is negative (to left)

(b) Equilibrium Position (b)  
 $x = 0, F = 0, a = 0$

(c) Compressed compared with (b)  
 $x$  is negative (to left)  
 $F$  is positive (to right)  
 $a$  is positive (to right)

A graph of acceleration vs Extension

The graph is linear since the acceleration is directly proportional to  $x$  (displacement)

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Definition of Simple Harmonic Motion

SHM is oscillatory motion in which the acceleration is:

- proportional to the displacement and
- is directed toward the equilibrium.

Since the acceleration is directly proportional and in the opposite direction, we can make a proportionality statement:

$a \propto -x$   
 $a = -\omega^2 x$  ← Defining Equation for SHM

where  $x$  is the displacement  
 $a$  is the acceleration  
 $\omega^2$  is the proportionality constant

$\omega$  is called the angular frequency of the oscillation (units:  $s^{-1}$  or  $rad\ s^{-1}$ )

In this case (i.e. bob attached to the horizontal spring)  $\omega$  is constant

$a = -\frac{k}{m}x$   
 $a = -\omega^2 x$   
 so  $\omega^2 = \frac{k}{m}$

Notes:  $\omega$  is constant  
 - We solve kinematics problems involving SHM using our "5 usual" equations because the acceleration is NOT constant during SHM. The acceleration is constantly changing!  
 - the significance of the kinematics graphs ( $x-t, v-t, a-t$ ) are still valid

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To show that an oscillatory motion IS simple harmonic motion, then we must show that:

Consider the pendulum:  
 It has oscillatory motion, but is it SHM??

$a \propto -x$

Restoring force

$F_{net} = mg \sin \theta$   
 $F_{net} = mg \frac{x}{l}$   
 $ma = \frac{mg}{l} x$   
 $a = \frac{g}{l} x$   
 insert -ve since acc is opp  $x$

$F_T \cos \theta = mg \cos \theta$   
 $F_T \sin \theta = mg \sin \theta$

Recall:  $a = -\omega^2 x$  ← since the acceleration of the pendulum fits the defining equation for SHM, then its oscillatory motion is, in fact, SHM.

$\therefore \omega^2 = \frac{g}{l}$

$v = \omega x \quad \omega = \frac{v}{x}$

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